AQA

A-LEVEL
Mathematics

MS03 – Statistics 3
Final Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS03

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

GN8 Where a candidate's response to a part of a question is simply to label the part (eg (d)(i)) with nothing else (ie no attempt at a solution), then this is still treated as a response and marked as 0 rather than NR. Also, deleted work, if not replaced, should be marked and not treated as NR.

Q	Solution	Mark	Total	Comment
1 (a)	**Alfie**; since $r < 0$ **or** negative	B1		Name & reason (OE)
	Chao; since $r > 1$ **or** incorrect **or** impossible	B1	**2**	Name & reason (OE)
(b)	CV (n = 12 & 5% 1-tailed) = **<u>0.497</u>**	B1		AWRT (0.4973)
	(Bishen) r = **<u>0.556 > CV</u>**	M1		Comparison of correct calculated r with tabulated r **or** value is significant Can be implied by conclusion
	Thus, at 5% level, **reject H_0** in favour of H_1 **or** **Evidence**, at 5% level, of a **positive correlation**	A1	**3**	OE; not definitive
		Total	**5**	

Q	Solution	Mark	Total	Comment
2	99% $\Rightarrow$ z = **<u>2.57 to 2.58</u>**	B1		AWFW (2.5758)
	CI for λ:			
	$\begin{pmatrix}1521\\304.2\\117\\23.4\end{pmatrix} \pm \begin{pmatrix}2.57 \text{ to } 2.58\\2.32 \text{ to } 2.33\end{pmatrix}\begin{pmatrix}\sqrt{1521}=39\\\sqrt{304.2/5}=7.8\\\sqrt{117/13}=3\\\sqrt{23.4/65}=0.6\end{pmatrix}$	M1		$\lambda \pm \left(2.5(7\text{to}8) \text{ or } 2.3(2\text{to}3)\right)\sqrt{a}$
		A1		Any correct value for λ
		A1		Correct expression for a given λ
	or			
	$\begin{pmatrix}1521\\304.2\\117\\23.4\end{pmatrix} \pm \begin{pmatrix}90.48 \text{ to } 100.62\\18.096 \text{ to } 20.124\\6.96 \text{ to } 7.74\\1.392 \text{ to } 1.548\end{pmatrix}$			or $\begin{pmatrix}1420 \text{ to } 1431, 1611 \text{ to } 1622\\284.0 \text{ to } 286.2, 322.2 \text{ to } 324.4\\109.2 \text{ to } 110.0, 124.0 \text{ to } 124.8\\21.8 \text{ to } 22.0, 24.8 \text{ to } 25.0\end{pmatrix}$
	Dividing by 65, 13, 5 or 1 as appropriate	A1		Can be implied
	<u>23.4 ± (1.5 to 1.6)</u> **or** **<u>(21.8 to 21.9, 24.9 to 25.0)</u>**	A1	**6**	CAO ± AWFW AWFW
		Total	**6**	

Q	Solution	Mark	Total	Comment
3(a) (i)	P(NC) =			
	$0.25 \times 0.015 + 0.35 \times 0.01 + 0.40 \times 0.005$ **or** $0.00375 + 0.0035 + 0.002$	M1		At least 2 terms correct
	= **0.00925 or 37/4000**	A1	**2**	CAO
(ii)	$P(C \mid NC) = \frac{0.40 \times 0.005}{(a)(i)}$ **or** $\frac{0.002}{(a)(i)}$	M1		Correct numerator ÷ [(a)(i) OE]
	= **8/37 or 0.216**	A1	**2**	CAO/AWRT (0.21622)
(b)	$P((A \cap B \cap C) \mid NC)$ =			
	$\frac{0.00375}{0.00925} \times \frac{0.0035}{0.00925} \times \frac{0.002}{0.00925}$ **or** $\frac{15 \times 14 \times 8}{37^3}$ **or** $\frac{1680}{50653}$ **or** 0.40541 × 0.37838 × 0.21622 **or** 0.03317	M1		Correct to 3sf
	× 6	A1		CAO
	= **0.198 to 0.199**	A1	**3**	AWFW (0.199001)
(c)	P(NC) =			
	$0.35 \times 0.01 + 0.65 \times 0.005$ **or** $0.0035 + 0.00325$	M1		Both terms correct
	= **0.00675 or 27/4000**	A1		CAO
	$P(C \mid NC) = \frac{0.65 \times 0.005}{0.00675}$ **or** $\frac{0.00325}{0.00675}$			Can score previous M1 A1 here
	= **13/27 or (0.481 to 0.482)**	A1	**3**	CAO/AWFW (0.481481)
		Total	**10**	

Q	Solution	Mark	Total	Comment
4				
(a)	$\bar{x}_{OS}$ = **<u>15.5</u>**	B1		CAO
	98% $\Rightarrow$ z = **<u>2.32 to 2.33</u>**	B1		AWFW (2.3263)
	CI for $\mu_{OS} - \mu_{UK}$ is:			
	$(15.5-10.8) \pm \begin{pmatrix}2.32 \text{ to } 2.33\\2.05 \text{ to } 2.06\end{pmatrix} \times \left(\sqrt{\frac{3.2^2}{25}+\frac{4.7^2}{16}}\right)$	M2 (–1 ee)		$\sqrt{1.790225}$ = 1.33799 Pooling is not valid but see SC(a)
	<u>4.7 ± 3.1</u> **or** **<u>(1.6, 7.8)</u>**	A1dep		CAO/AWRT (3.11257) Dep on M2 AWRT
			5	
SC(a)	**1** CI is: $(15.5-10.8) \pm \begin{pmatrix}2.32 \text{ to } 2.33\\2.05 \text{ to } 2.06\end{pmatrix} \times \left(\sqrt{\frac{24\times3.2^2+15\times4.7^2}{39}\left(\frac{1}{25}+\frac{1}{16}\right)}\right)$ = 4.7 ± 2.9 = (1.8, 7.6) $\Rightarrow$ B1 B1 M1 A0dep (max of 3 marks)			
(b)	Since 0 < LCL **or** 0 $\notin$ CI	Bdep1		Dep on LCI > 0
	Evidence, at 2% level, that $\mu_{OS} \neq \mu_{UK}$ **or** **Evidence**, at 1% level, that $\mu_{OS} > \mu_{UK}$	Bdep1		Dep on Bdep1; not definitive
			2	
(c)	$2 \times \begin{pmatrix}2.32 \text{ to } 2.33\\2.05 \text{ to } 2.06\end{pmatrix} \times \sqrt{\frac{3.2^2}{n}+\frac{4.7^2}{n}} = (\text{or} <) 2$	M1 A1		OE; allow no '2' on LHS If pooling used then see SC(b) Fully correct
	$n (= \text{or} >) (2.32 \text{ to } 2.33)^2 \times (32.33)$	m1		Correct method for solving for n
	n = **<u>175</u>** **or** n = **<u>180</u>**	A1		If n < 175 CAO; either If n > 175
			4	
SC(b)	**1** Equation/inequality for n is: $2 \times \begin{pmatrix}2.32 \text{ to } 2.33\\2.05 \text{ to } 2.06\end{pmatrix} \times \left(\sqrt{\frac{24\times3.2^2+15\times4.7^2}{39}\left(\frac{1}{n}+\frac{1}{n}\right)}\right) = (\text{or} <) 2$ $\Rightarrow$ n = 160 or 165 $\Rightarrow$ M1 A0 m1 A0 (max of 2 marks)			
		Total	**11**	

Q	Solution	Mark	Total	Comment
5(a) **(i)**	Mean, $\mu_M = 30 + 15 = \underline{\mathbf{45}}$	B1		CAO
	Variance, $\sigma_M^2 = 25 + 9 + \mathbf{2 \times 0.8 \times 5 \times 3}$ $= 34 + 24$	M1		Use of $\sigma_V^2 + \sigma_W^2 + 2\rho_{VW}\sigma_V\sigma_W$ (OE)
	$= \underline{\mathbf{58}}$	A1	**3**	CAO
(ii)	Mean, $\mu_D = 45 - (5 + 25 + 10) = \underline{\mathbf{5}}$	BF1		F on [(i) – 45]
	Variance, $\sigma_D^2 = 58 + (1 + 16 + 4) = \underline{\mathbf{79}}$	BF1	**2**	F on [(i) + 21]
(b) **(i)**	Require $P(M > 60) = P\left(Z > \frac{60-45}{\sqrt{58}}\right)$	M1		Standardising **60** with c's $\left(\mu_M \text{ and } \sqrt{\sigma_M^2}\right)$ from (a)(i)
	$= P(Z > 1.9696) = 1 - P(Z < 1.9696)$			
	$= \underline{\mathbf{0.024 \text{ to } 0.025}}$	A1	**(2)**	AWFW (0.02444)
(ii)	Require $P(M < (U + X + Y)) = \underline{\mathbf{P(D < 0)}}$	M1		Correct change to D
	$= P\left(Z < \frac{0-5}{\sqrt{79}}\right) = P(Z < (\underline{\mathbf{-0.563 \text{ to } -0.562}})$	A1		AWFW; ignore sign
	$= \underline{\mathbf{0.286 \text{ to } 0.288}}$	A1	**(3)**	AWFW (0.286873)
			5	
		Total	**10**	

Q	Solution	Mark	Total	Comment
6(a)(i)	$\mathrm{E}\big(X\,(X-1)\big) = \sum_{x=0}^{n} x(x-1)\left(\frac{n!}{x!(n-x)!}\right)p^x(1-p)^{n-x}$	M1		Allow $\binom{n}{x}$; ignore limits
	$= n(n-1)p^2\sum_{x=2}^{n}\left(\frac{(n-2)!}{(x-2)!(n-x)!}\right)p^{x-2}(1-p)^{n-x}$ $= n(n-1)p^2\sum_{y=0}^{m}\left(\frac{(m)!}{(y)!(m-y)!}\right)p^{y}(1-p)^{m-y} =$	m1		Factor of $n(n-1)p^2$, substituting $y = x-2$ and $m = n-2$, using $\sum \mathrm{B}(m,p) = 1$ is required; do **not** ignore y/m but ignore limits
	$\boldsymbol{n(n-1)p^2}$	A1		Clear fully correct derivation
	$\mathrm{Var}(X) = \mathrm{E}\big(X\,(X-1)\big) + \mathrm{E}(X) - \big(\mathrm{E}(X)\big)^2 =$	M1		Used for B(n, p)
	$n^2p^2 - np^2 + np - n^2p^2 = \underline{\boldsymbol{np(1-p)}}$	A1	**5**	**AG**: clear fully correct deduction
Note	**1** Other valid derivations are possible and acceptable			
(ii)	$\frac{\mathrm{P}(X=x)}{\mathrm{P}(X=x-1)} = \left(\frac{n!}{x!(n-x)!}\right)\left(\frac{(x-1)!(n-x+1)!}{n!}\right)\left(\frac{p^x(1-p)^{n-x}}{p^{x-1}(1-p)^{n-x+1}}\right)$	M1		Attempted ratio OE; eg factorise $\mathrm{P}(X=x)$ to show as 'answer' × $\mathrm{P}(X=x-1)$
	$= \underline{\left(\frac{\boldsymbol{n-x+1}}{\boldsymbol{x}}\right)\left(\frac{\boldsymbol{p}}{\boldsymbol{1-p}}\right)}$	A1	**2**	**AG**: clear fully correct deduction
(b)	$\left(\frac{n-7}{8}\right)\left(\frac{p}{1-p}\right) = 1 \;\Rightarrow\; np = 8-p$	M1		Symmetry of P(7) & P(8) wrt E(Y) implies $p = 0.5 \Rightarrow$ B2
	But $\mathrm{E}(Y) = 7.5 = np$ $\Rightarrow$ p = **<u>0.5</u>**	A1		CAO; implied by 3.75
	Thus $\mathrm{Var}(Y) = 7.5 \times 0.5 =$ **<u>3.75</u>**	B1	**3**	CAO
(c)	$\frac{\mathrm{Var}(U)}{\mathrm{E}(U)} = \frac{np(1-p)}{np} = \frac{4.995}{5}$ $\Rightarrow$ p = 0.001 and n = 5000	M1		OE; implied by Po(5)
	Thus n large and p small $\Rightarrow$ **<u>Po(5)</u>**	A1		Accept $(\mu \approx \sigma^2)$ or (rare event)
	$\mathrm{P}(U \le 3)$ = **<u>0.265</u>**	A1	**3**	AWRT (0.265026)
(d)	$\left(\frac{n-60}{61}\right)\left(\frac{p}{1-p}\right) = 1 \;\Rightarrow\; np = 61-p$			Or symmetry
	But $\mathrm{E}(V) = 60.5 = np$ $\Rightarrow$ **<u>p = 0.5 and n = 121</u>** **or** **<u>Var(V) = 30.25</u>**	B1		CAO; both CAO
	$\mathrm{P}(V > 50) = \mathrm{P}\left(Z > \frac{(50+0.5)-60.5}{\sqrt{30.25}}\right)$	M1		Normal used; allow 50 or 49.5
	$= \mathrm{P}(Z > -1.8182) =$ **<u>0.964 to 0.966</u>**	A1	**3**	AWFW (0.965482)
		Total	**16**	

Q	Solution	Mark	Total	Comment
7				
(a)	$H_0 : p = 0.25$ $H_1 : p > 0.25$	B1 B1		Accept 25%; both marks can be scored here or in (b)(i)
	$P(X \geq 10 \mid B(30, 0.25)$ = 1 – (0.8034 or 0.8943)	M1 m1		Use of B(30, 0.25); can be implied
	= **0.196 to 0.20**	A1		AWFW (0.196593)
	No evidence, at 5% level, **to support the claim**	Adep1	**6**	**AG** Dep on A1; not definitive
(b)				
(i)	5% $\Rightarrow$ z = **1.64 to 1.65**	B1		AWFW (1.6449)
	$z = \dfrac{\frac{90}{300} - 0.25}{\sqrt{\frac{0.25 \times 0.75}{300}}}$ **or** $z = \dfrac{(90 \pm 0.5) - 75}{\sqrt{56.25}}$	M1		
	= **2** **or** = **1.93 to 2.07** (0.02275) **or** (0.02688 to 0.01923)	A1		CAO/AWFW
	Evidence, at 5% level, **to support the claim**	Adep1	**4**	Dep on A1; not definitive
(ii)	$\dfrac{\hat{p} - 0.25}{\sqrt{\frac{0.25 \times 0.75}{300}}} = 1.64$ to 1.65	M1		
	$\hat{p}$ = **0.291**(1)	A1	**2**	**AG**; seen to ≥4dp (0.291123)
(iii)	Power = 1 – P(Type II error) or 1 – P(accept $H_0 \mid H_0$ false) or P(reject $H_0 \mid H_0$ false) or P(accept $H_1 \mid H_1$ true)	B1		Any one stated or used
	$P\left(\hat{P} > 0.291 \mid B(300, 0.30)\right)$ =	M1		Use of B(300, 0.30) ; can be implied
	$P\left(Z > \dfrac{0.291 - 0.30}{\sqrt{\frac{0.30 \times 0.70}{300}}}\right)$ **or** $P\left(Z > \dfrac{87.3 - 90}{\sqrt{63}}\right)$	m1		Must use p = 0.30 in denominator
	= P($Z >$ **–0.340**)	A1		AWRT; ignore sign (–0.340168)
	= **0.63 to 0.635**	A1	**5**	AWFW (0.633135)
		Total	**17**	